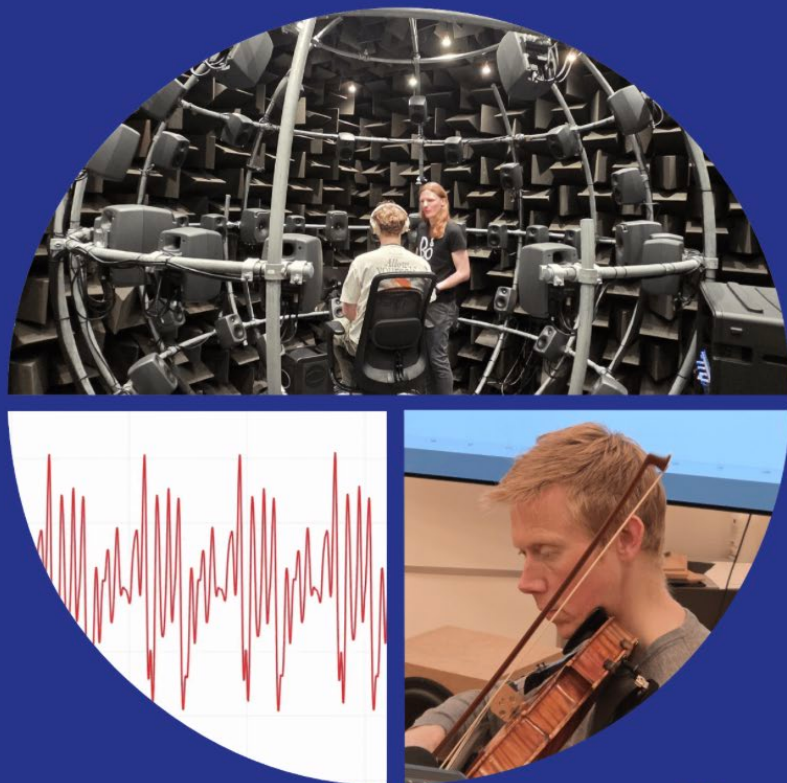


Matematikken bag den store lyd hos B&O



B&O

Mikkel Rønne
Bjørn Grøn



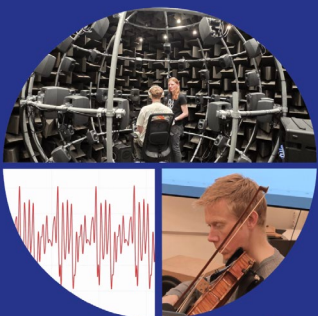
matematik
i arbejde

BJØRN GRØN (RED.), MADS LETH

Træk virksomhederne ind i undervisningen
November 2025



matematik
i arbejde



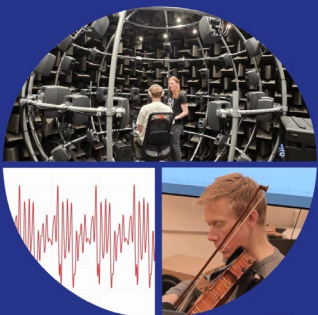
matematik
i arbejde

Indholdsfortegnelse

0. Indledning
1. Overtoner, musikinstrumenter og frekvensspektre
2. Fourierrækker
3. Stødtoner
4. Modellering med trigonometriske funktioner (tidevand)
5. Sampling og det binære talsystem
6. Filtre og støjreduktion
7. Måling af lyd med PhyPhox
8. Musik og matematik – Talteori
9. Musik og matematik – trigonometri og logaritmer



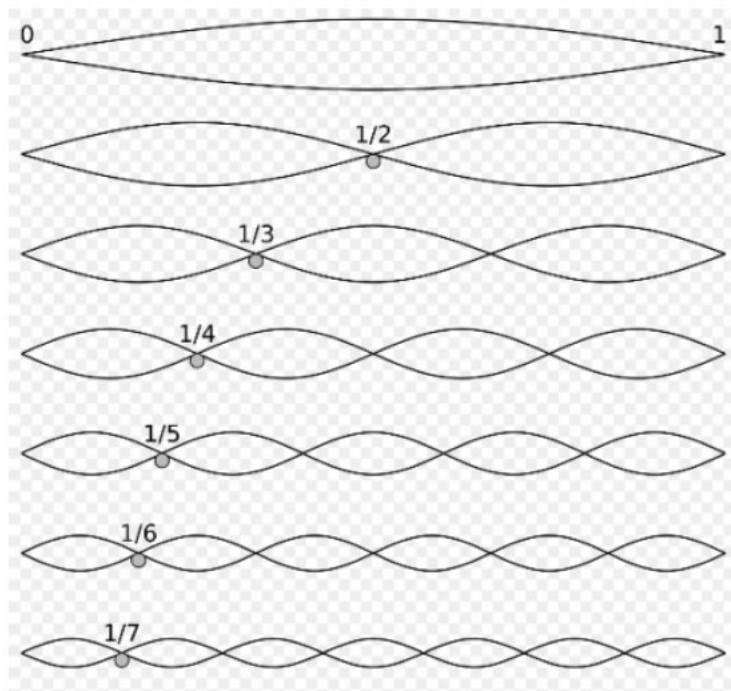
matematik
i arbejde



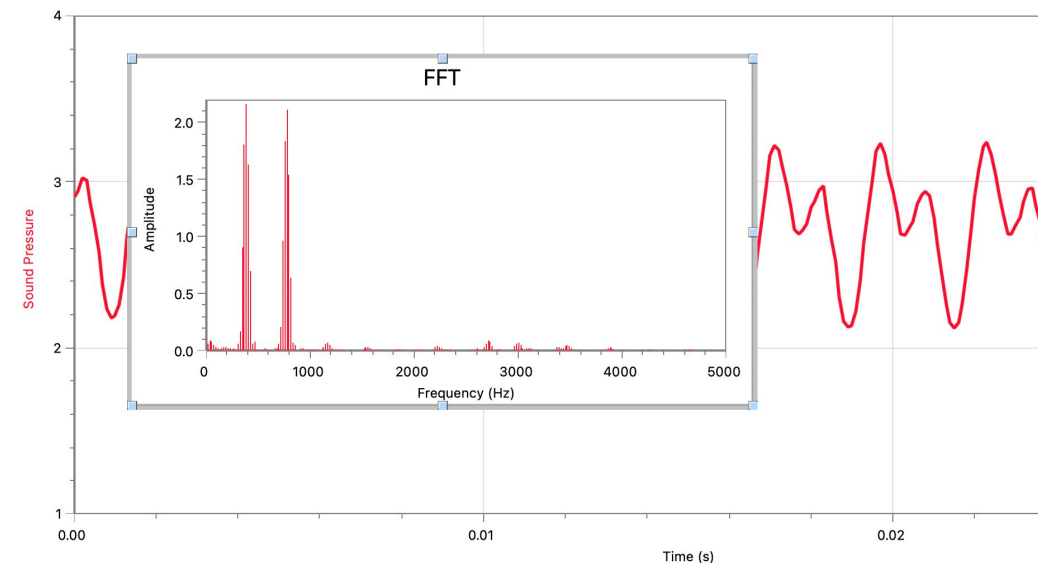
Overtoner og musikinstrumenter

Frekvensspektre

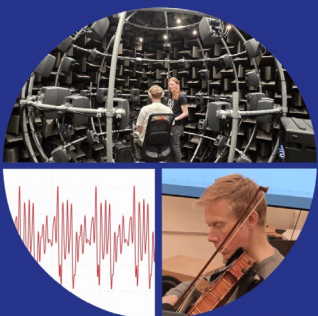
Stående bølger



Eksempel på en mennesketone analyseret med
FFT (Fast Fourier Transform) - LoggerPro

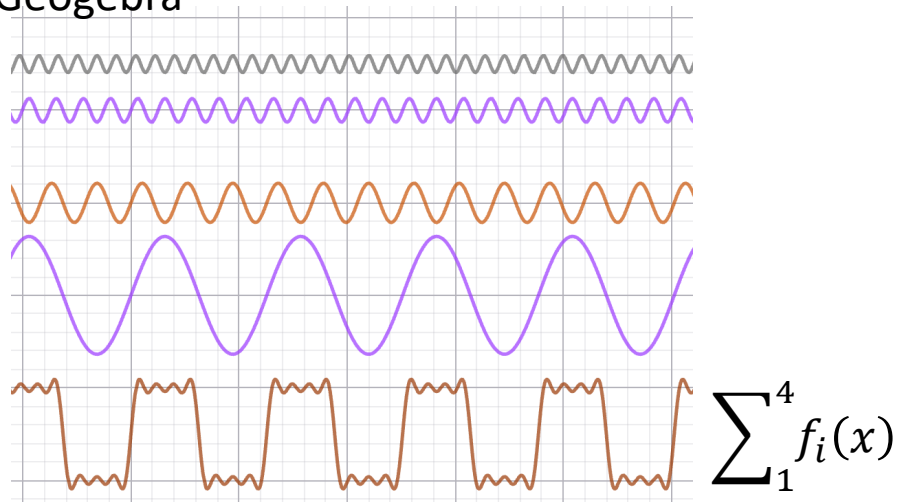


Materiale: s.23-24, s.63-73
Film: Overtoner 11.45-16.53,
Frekvensspektrum: 16.53-18.09



Fourierrækker

Geogebra

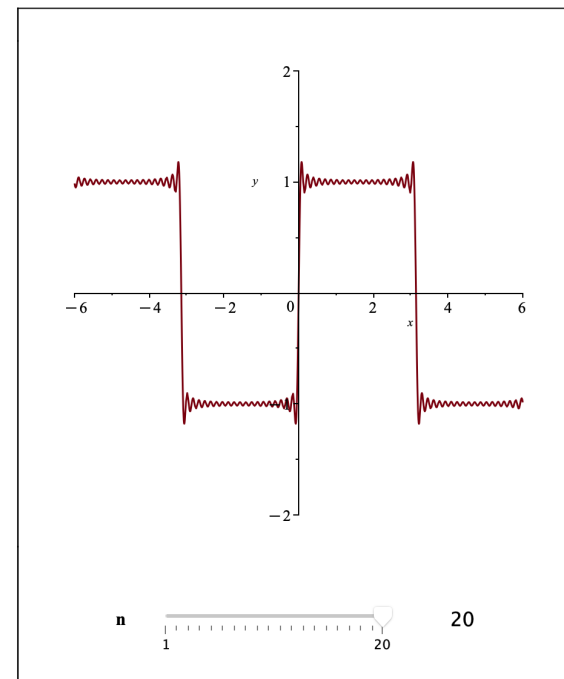


Materiale: s.62-73

$$f_n(x) = \frac{4}{\pi \cdot (2n + 1)} \cdot \sin((2n + 1)x)$$

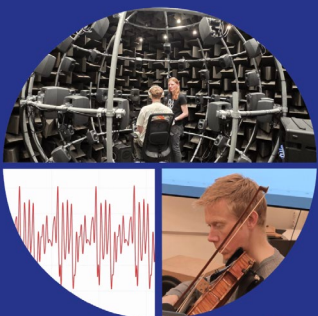
`Explore(plot(4/pi * sum(1/(2*i-1) * sin((2*i-1)*x)), x=-6..6, y=-2..2), parameters=[n=1..20])`

Maple



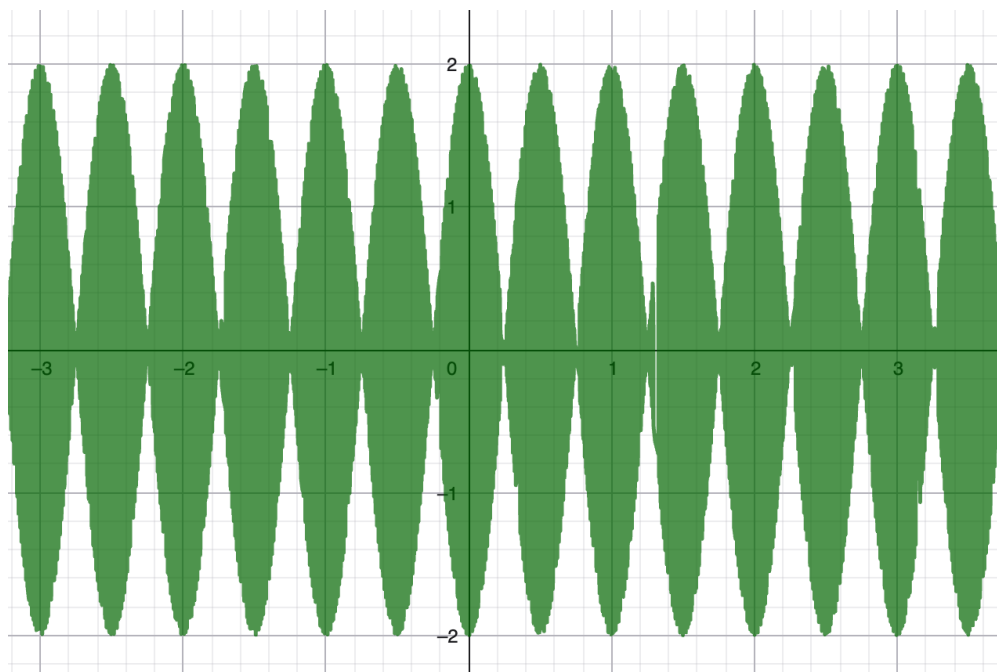
$$\sum_1^{20} f_i(x)$$





Stødtoner

$$f(x) = \sin(2\pi \cdot 440x) + \sin(2\pi \cdot 442x)$$



Additionsformler

$$\sin(u + v) = \sin(u) \cdot \cos(v) + \cos(u) \cdot \sin(v)$$

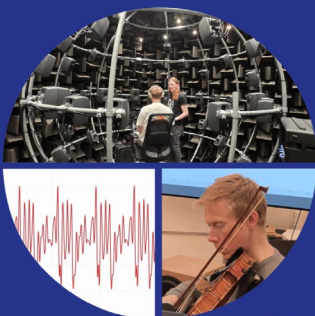
$$\sin(u - v) = \sin(u) \cdot \cos(v) - \cos(u) \cdot \sin(v)$$

Logaritmiske formler

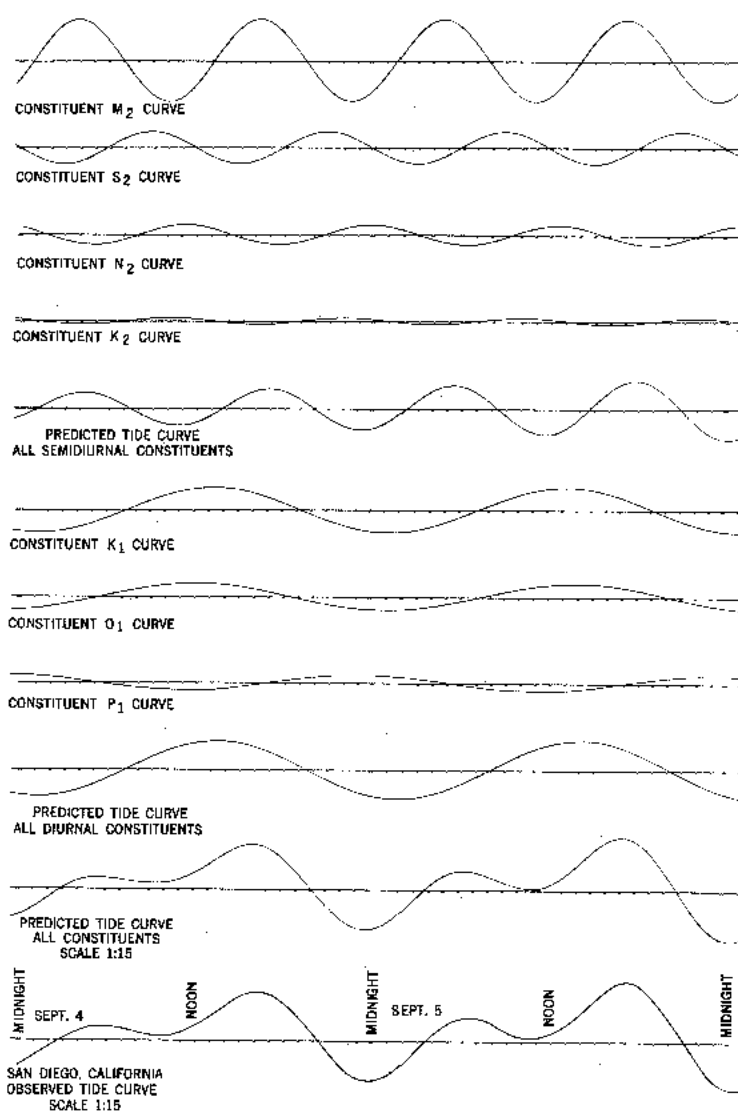
$$\sin(x) + \sin(y) = 2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

Materiale: s.25-28





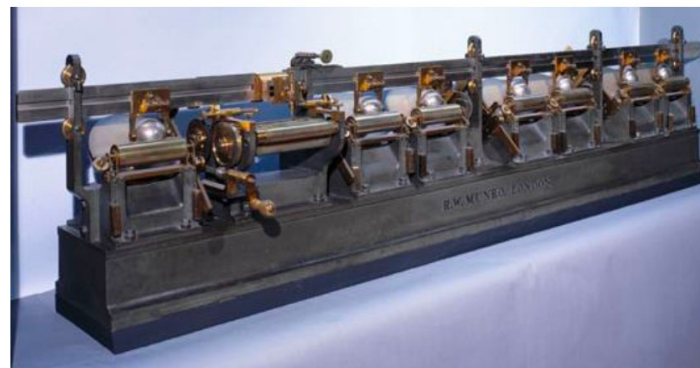
Modellering med trigonometriske funktioner



Tidevand



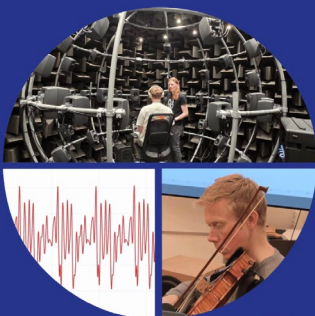
Højvande og lavvande i Port St. Ives, Cornwall, England.



Kelvins harmoniske analysator fra 1878. Den opbevares i dag på
Science-museet i London.

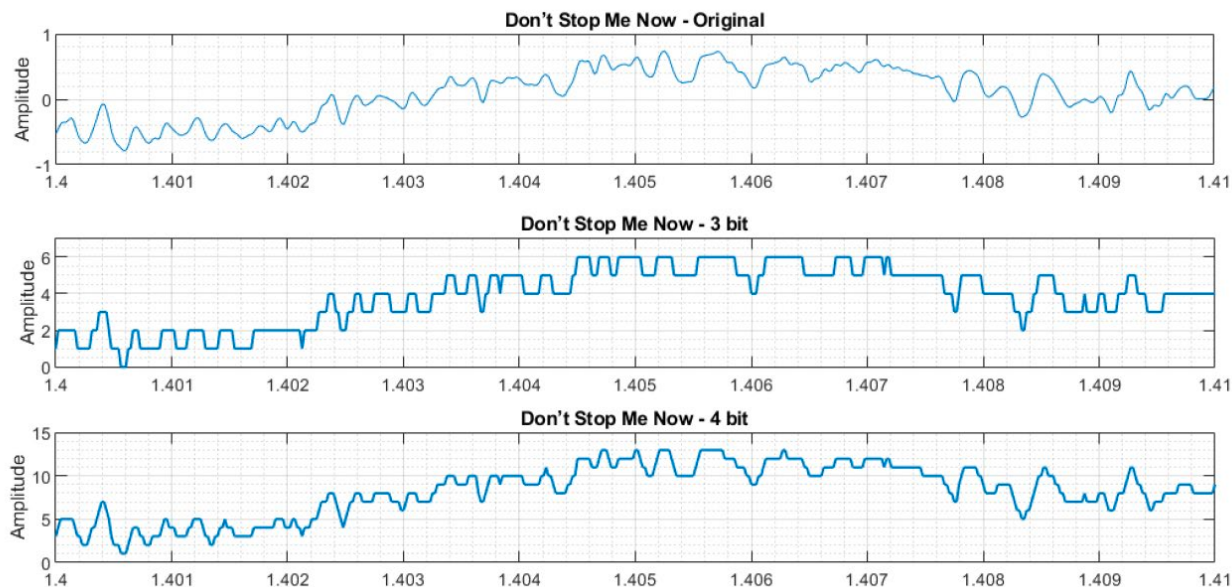
Materiale: s.50-61

Film: Modellering af lyd 6.35-11.45



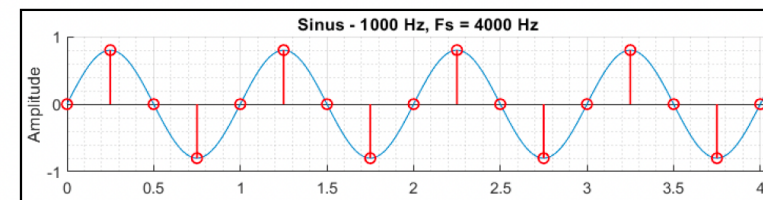
matematik i arbejde

Sampling og det binære talsystem



Position	7	6	5	4	3	2	1
Værdi	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Tæller antal...	64'ere	32'ere	16'ere	8'ere	4'ere	2'ere	1'ere

Nyquist-teoremet

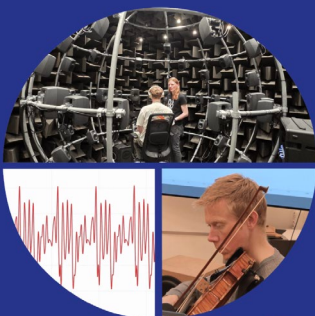


Materiale: s.74-89
Film: 21.26-34.26



matematik i arbejde

Matematikken bag den store lyd hos B&O



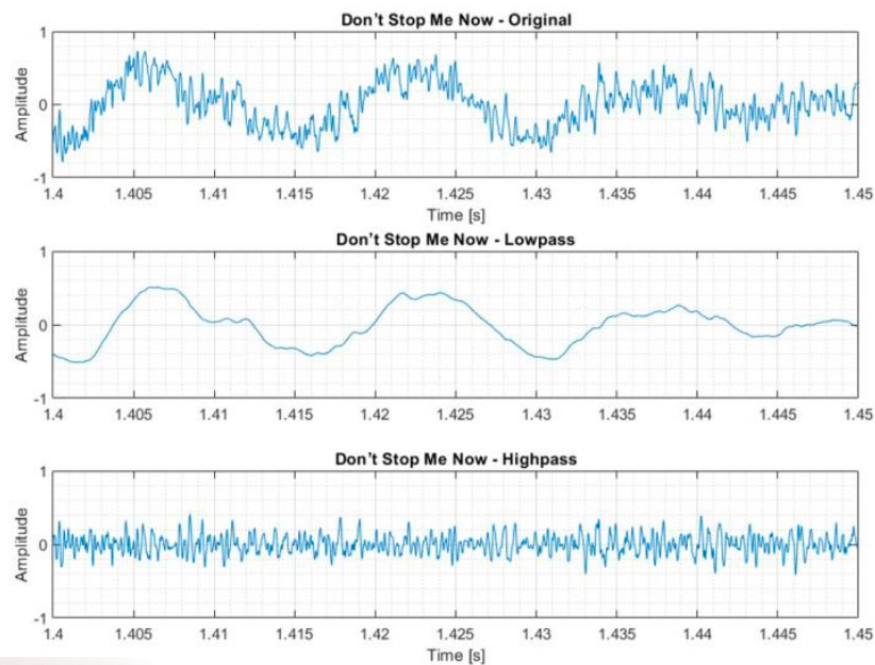
matematik
i arbejde

Bjarne Grøn (red.), Mads Leth

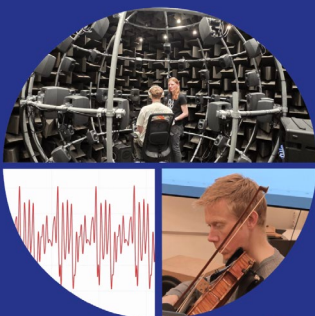


matematik
i arbejde

Filtre og støjreduktion

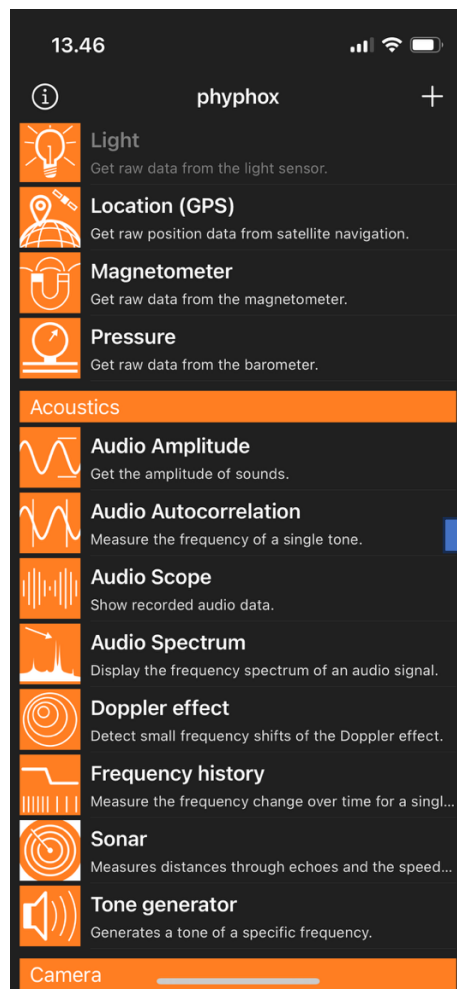


Materiale: s.90-96
Film: 34.26-



Måling af lyd med mobiltelefonen

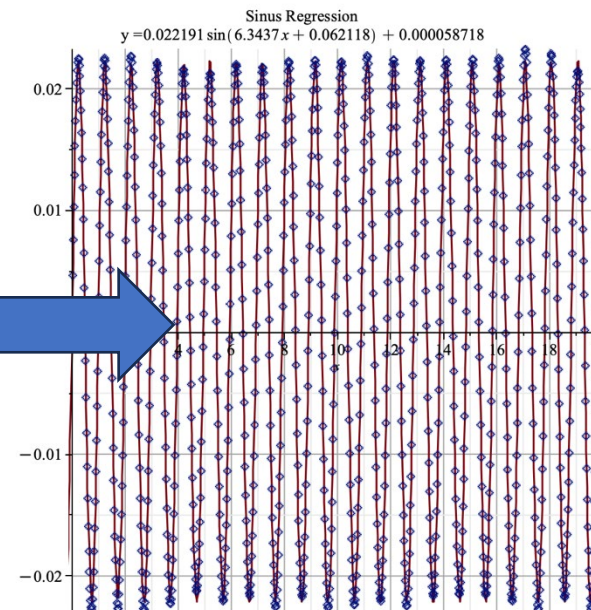
PhyPhox



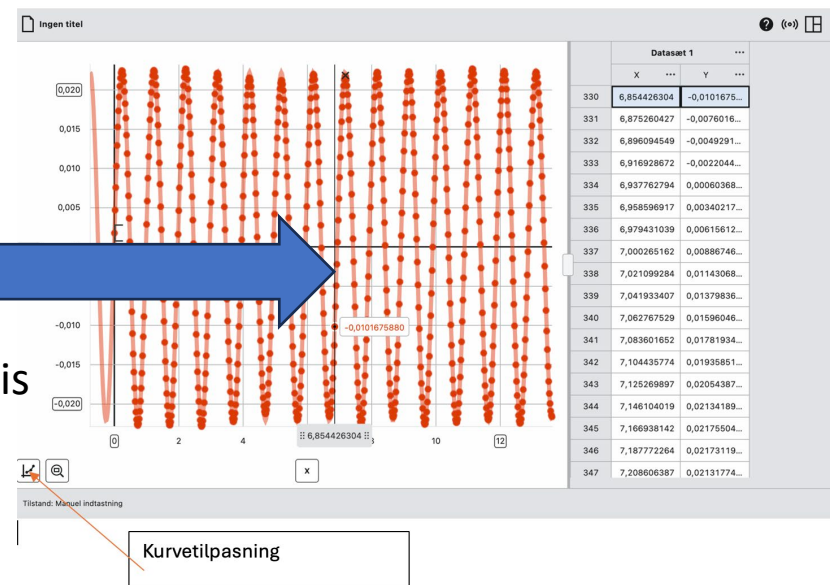
Excel

	A	B	C
1	Time (ms)	Recording (a.u.)	
2	0	0,00176951	
3	0,02083412	0,00469941	
4	0,04166825	0,00757328	
5	0,06250237	0,01030154	
6	0,08333649	0,01293262	
7	0,10417061	0,01529124	
8	0,12500474	0,01733955	
9	0,14583886	0,01911814	
10	0,16667298	0,02048767	
11	0,1875071	0,02152194	
12	0,20834123	0,02218194	
13	0,22917535	0,02247303	
14	0,25000947	0,02240852	
15	0,27084359	0,02190944	
16	0,29167771	0,02106314	
17	0,31251184	0,01981339	
18	0,33334596	0,01825487	
19	0,35418008	0,01679635	

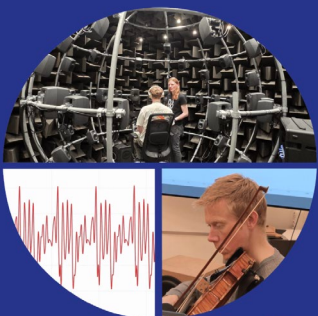
Maple



Vernier: Graphical Analysis

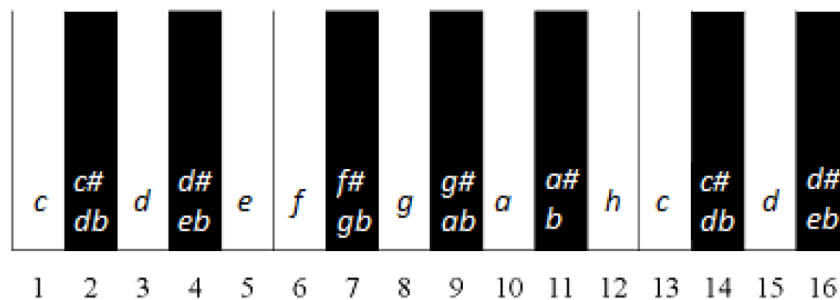


Materiale: Findes ikke i materiale

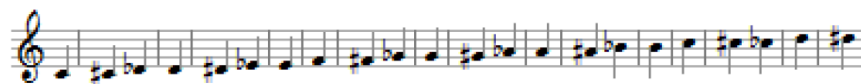


Matematik og musik: Talteori, når frekvensen fordobles

Hvordan stemmes de 12 toner indbyrdes



$$a^{12} = 2 ?$$



frekvenserne er bestemt af bølgelængden

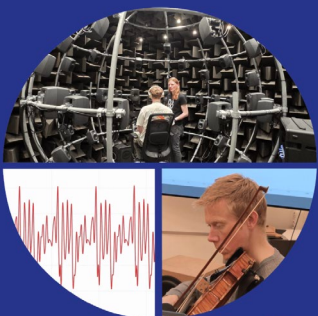
De <i>rene</i> *) intervaller	Strengelængde L = hele strengen	Frekvensforhold
Oktaven	$\frac{1}{2} \cdot L$	2
Kvint	$\frac{2}{3} \cdot L$	$\frac{3}{2}$
Kvarten	$\frac{3}{4} \cdot L$	$\frac{4}{3}$
Stortertsen	$\frac{4}{5} \cdot L$	$\frac{5}{4}$
Lilletertsen	$\frac{5}{6} \cdot L$	$\frac{6}{5}$

<https://upload.wikimedia.org/score/2/c/2cta7in0ywqe9ht097nes0607drdakq/2cta7in0.mp3>

Materiale: s.32-48

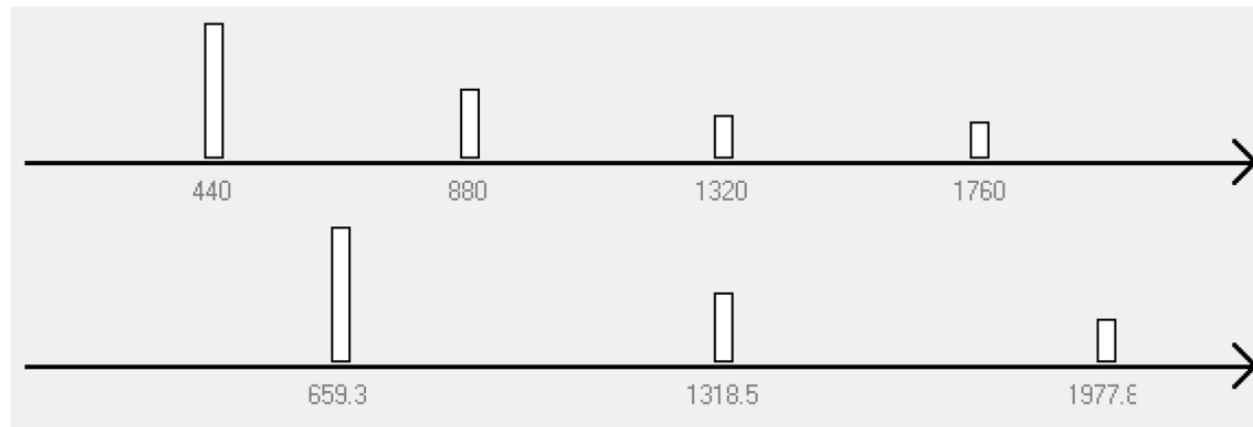
Film: 21.26-34.26





Matematik og musik: Trigonometri og logaritmer

Kvinten og oktaven? Svævning



Materiale: s.32-48

Film: 21.26-34.26

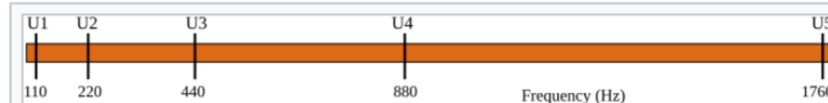
Tonen *a* har partialtonerne 440 Hz - 880 Hz - 1320 Hz - 1760 Hz - 2200 Hz ...

Tonen *e* har partialtonerne 659.3 Hz - 1318.5 Hz - 1977.8 Hz - 2637.0 Hz ...

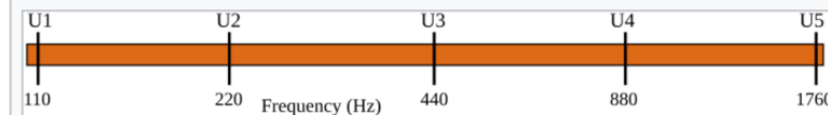
Cent-funktionen

$$\text{cent}\left(\frac{f_1}{f_2}\right) = 1200 \cdot \frac{\log_{10}\left(\frac{f_1}{f_2}\right)}{\log_{10}(2)}$$

$$\text{cent}\left(\frac{3}{2}\right) = 1200 \cdot \frac{\log_{10}\left(\frac{3}{2}\right)}{\log_{10}(2)} = 702$$



Octaves increase exponentially when measured on a linear frequency scale (Hz).



Octaves are equally spaced when measured on a logarithmic scale (cents).